Unraveling Extra Dimensions

(And Why You Want To)

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Outline

- History
  - Old Lore: 1910's-1980's
    - Unification
    - Ed Witten Ruins Everything
    - (Super)String Theory
  - Modern Canon: 1990's-
    - Large Extra Dimensions
    - Warping
    - Duality with Technicolor
    - Universal Extra Dimensions
- Phenomenology
  -
    - Dark Matter
    - Excited Modes
    - Black Holes

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The KK Idea: A Simple Picture

- (Due to Nordström, Kaluza, & Klein)
- Consider a 1-D problem, say, an ant crawling along a string.
- Even if all ants are given the same energy, some may traverse the length of the string at different rates, thus appearing to have different masses... Why?

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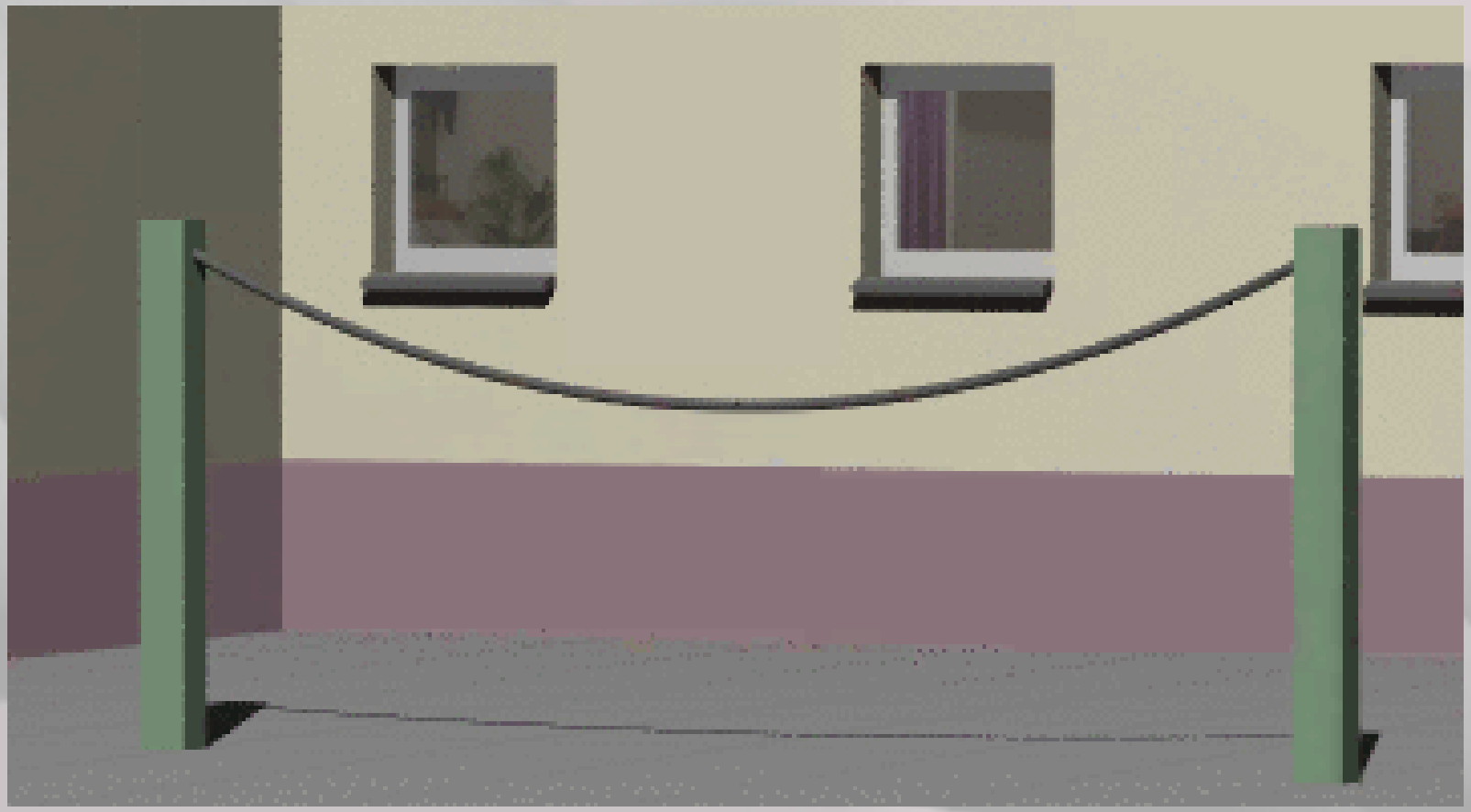

Figure: 1-D Problem.

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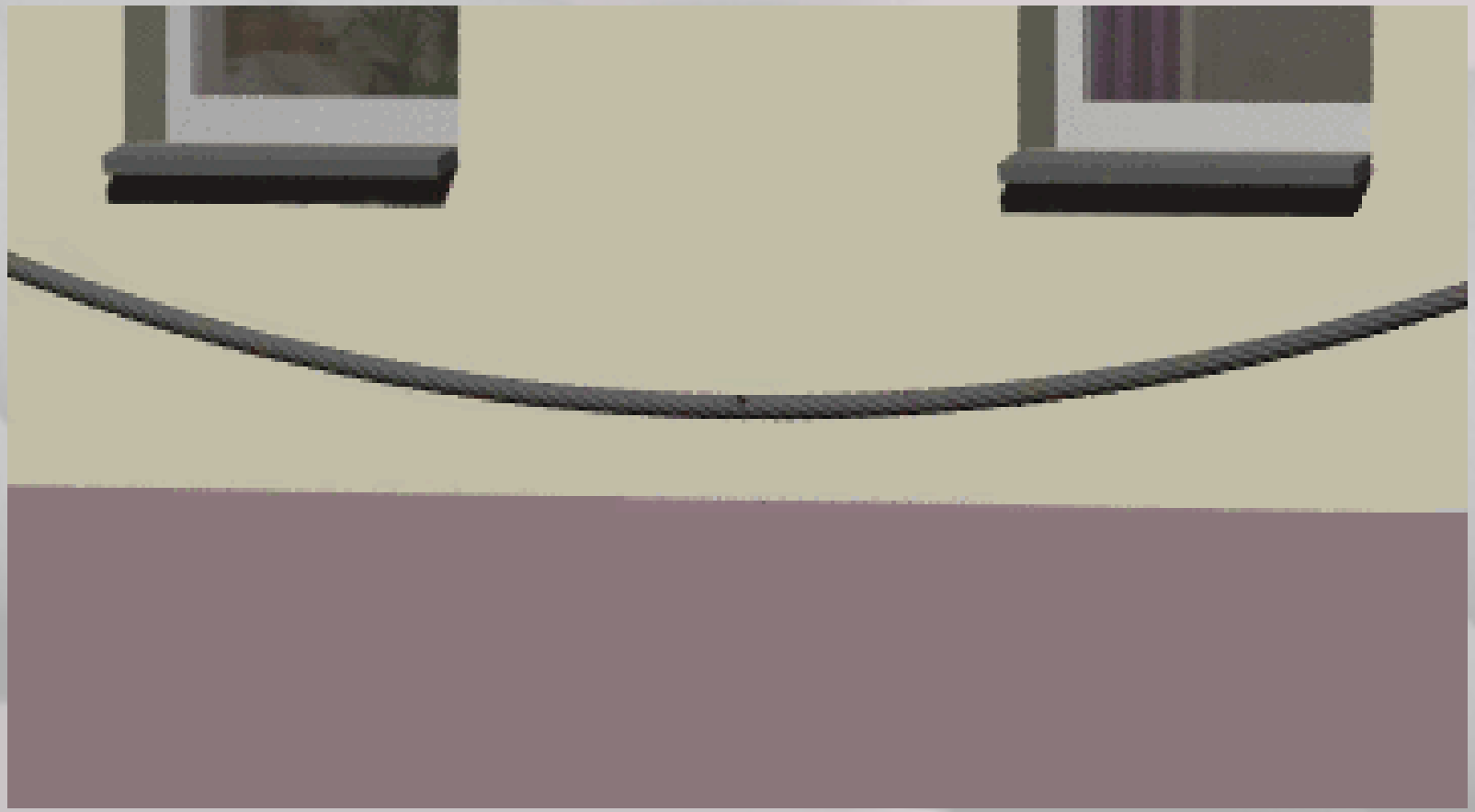

Figure: 1-D Problem...

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Figure: 1-D Problem?

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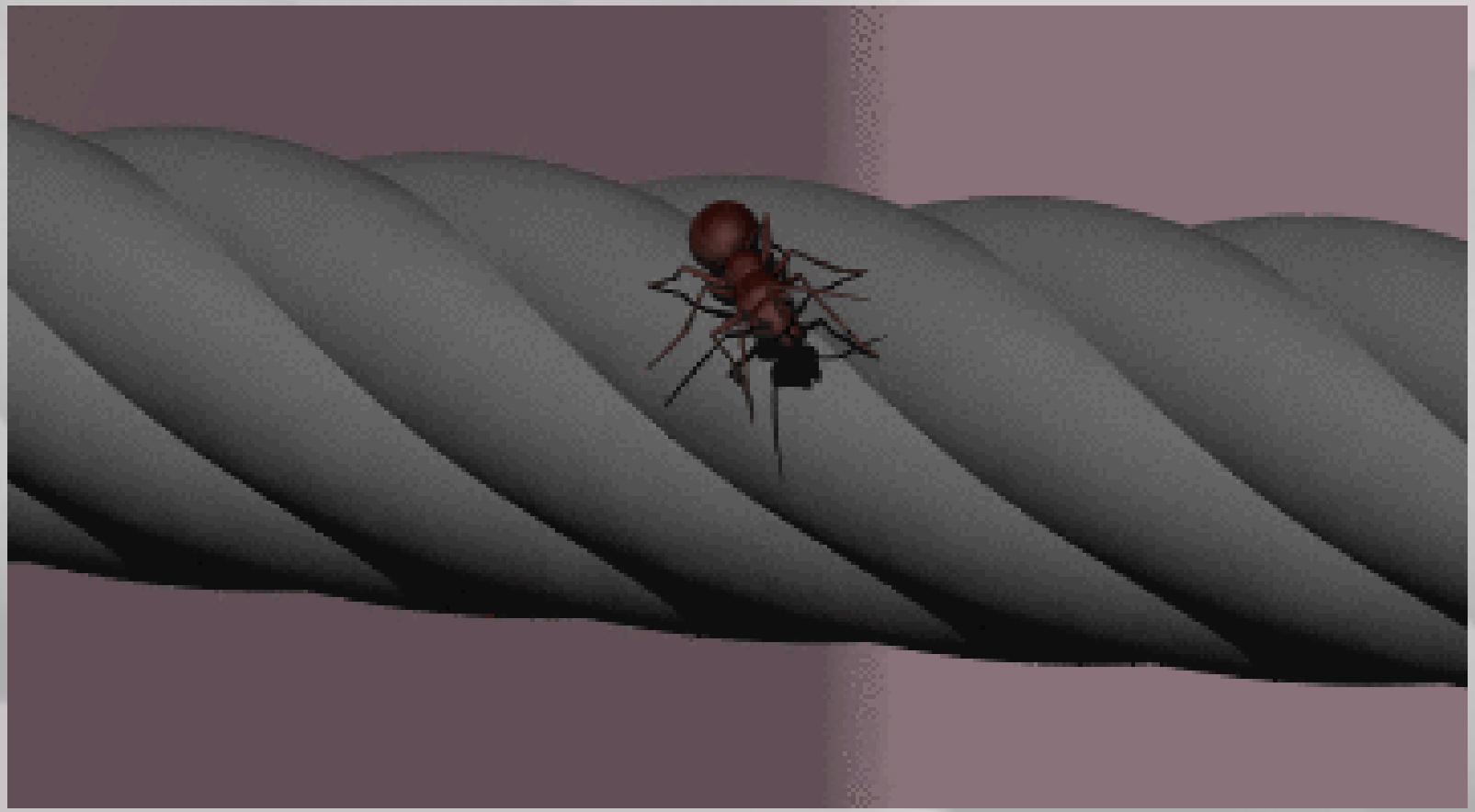

Figure: 2-D Problem?

A Simple Picture

Figure: 2-D Problem.

A Less Simple Picture

- Now picture doing the same thing to a 2-D or 3-D problem.
- We can't.
  - $S^1 \times \mathbb{R}^n$ is not emeddable in $\mathbb{R}^3$ for $n > 1$.
  - Nontrivial fiber bundles (twisting)
- Our macroscopic intuitions v. mathematical consistency of a theory

$\Rightarrow$ Our first motivation for (microscopic) extra dimensions:
Why not?

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Extra Dimensions in GR

The Real Fun — GSW, **hep-th/9410046**

- Kaluza: Consider a 5-D metric $\mathfrak{g}_{MN}$ and define $\phi = -\frac{1}{\sqrt{3}} \log \mathfrak{g}_{44}$, $A_\mu = \frac{\mathfrak{g}_{4\mu}}{\mathfrak{g}_{44}}$, and $g_{\mu\nu} = \mathfrak{g}_{\mu\nu} - \mathfrak{g}_{44} A_\mu A_\nu$.
- The 5-D generalization of the Einstein-Hilbert action is

$$S = \frac{1}{2\mathfrak{K}^2} \int \mathfrak{R} \sqrt{|\mathfrak{g}|} d^5\mathfrak{x}$$

- If for some reason $\mathfrak{g}_{MN}$ is independent of $\mathfrak{x}^4$, then the equations of motion simplify dramatically and (after a bit of rescaling), we find
  - A^μ obeys Maxwell's equations,
  - ϕ obeys the massless Klein-Gordon equation, and
  - $g_{\mu\nu}$ obeys the (4-D) Einstein equation.
- This is exciting - E&M and 4-D GR seem to have emerged from 5-D GR...

Compactification

Leads to Unification

...but it was hardly "natural" to assume one dimension just didn't matter.

- Recall our friend, the ant.
- Klein: If the fifth dimension is compact (say, a small circle of radius R), the momentum in that direction is quantized: $\phi(\mathfrak{x}) = \sum_n \phi_n(x) e^{in\mathfrak{x}^4/R}$, and similarly for A^μ and $g_{\mu\nu}$.
- The $n = 0$ modes have no momentum in the 5^{th} dimension, but the $n > 0$ modes have $\mathfrak{p}^4 = \frac{n}{R}$.
- If you're too big to know about the 5^{th} dimension, $E^2 = p^2 + (\mathfrak{p}^4)^2$ looks a lot like $E^2 = p^2 + m^2$.

$\Rightarrow$ At energies $E \ll \frac{1}{R}$, we get 4-D GR and E&M, all from GR on $\mathbb{R}^{(3,1)} \times S^1$. (**Unification**!)

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Nothing Good Lasts Forever...

- Where is the ϕ_0? Actually not as massless as we thought...
- Nature's more complicated than just E&M, but more dimensions can get bigger gauge groups.
- It's difficult to deal with fermions in some numbers of dimensions.
- In 1957, Wu, Ambler, Hayward, Hoppes, Hudson, Garwin, Lederman, and Weinrich ruined how pretty nature is and proved that the weak interaction (maximally) violates parity.
- In 1981, Witten proved that no way can KK generate our parity-violating gauge group.

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~~Unification~~!

String Theory

GSW

- Naïvely, quantum gravity is non-renormalizable.
  - Most understood solution: string theory
    - Needs conformal symmetry
    - For conformal symmetry to be quantum-mechanically consistent, needs anomalies to cancel
    - Cancels iff $D = 26$ (bosonic strings only)
      ^(theory appears inconsistent)
    - Needs $D = 10$ for superstrings

$\Rightarrow$ **Quantizing gravity consistently** **may** *require* extra dimensions.

10-D Geometry

- Naturally described in terms of orbifolds, getting around Witten's theorem
- ...**Unification**...?

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- Spacetime is an orbifold of $\mathbb{R}^{(3,1)} \times K^6$, where K^6 is something like

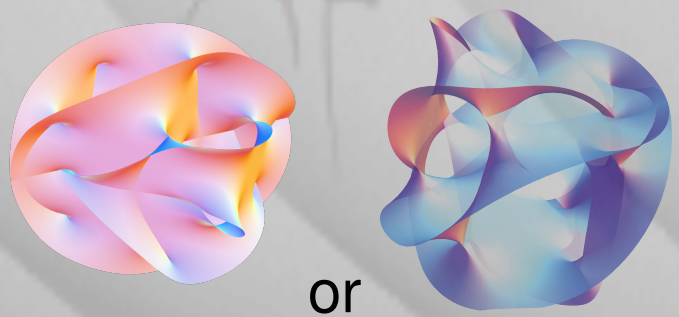

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Figure: Calabi-Yau Manifolds

- $\sim 10^{\sim 500}$ ways of doing this...
  Along with anthropic reasoning...
  $\Rightarrow$ **solve fine-tuning problems**...?

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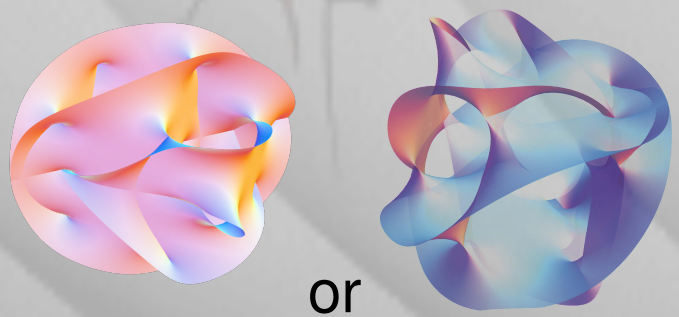

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- Large Extra Dimensions
- Warping
- Duality with Technicolor
- Universal Extra Dimensions

Phenomenology

- Dark Matter
- Excited Modes
- Black Holes

Large Extra Dimensions

(LED)

- hep-th/990522: Arkani-Hamed, Dimopoulos, & Dvali (ADD)
- In $4 + n$ dimensions, $V_g \sim \frac{m_1 m_2}{\mathfrak{M}_{\rm Pl}^{n+2} r^{n+1}}$.
- If extra dimensions compact, ^only true for $r \ll R$
- For $r \gg R$, get $V_g \sim \frac{m_1 m_2}{\mathfrak{M}_{\rm Pl}^{n+2} R^n r}$
- Equating this with our well known $V_g \sim \frac{m_1 m_2}{M_{\rm Pl}^2 r}$, we must conclude that our measured $M_{\rm Pl} \sim \mathfrak{M}_{\rm Pl} \left(\frac{\mathfrak{M}_{\rm Pl}}{R}\right)^{n/2}$.
- **"Solves" Hierarchy Problem**!
- $V_g \sim \frac{1}{r}$ only tested down to ~ 1 mm...

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Large Extra Dimensions

Problems

- I can see 1 mm! Things look $3+1$-dimensional
  - Demand SM stuck in a 4-D subspace of spacetime (membrane) but gravity propagates in bulk
- Since 1998: much more stringent tests on $V_g \sim \frac{1}{r}$
- $R\mathfrak{M}_{\mathrm{Pl}} = \left(\frac{M_{\mathrm{Pl}}}{\mathfrak{M}_{\mathrm{Pl}}}\right)^{2/n}$ - still a hierarchy!
- Ruins protection of SM as an effective theory from higher-dimensional operators

Warping

Compact

- hep-ph/9905221: Randall & Sundrum (RS1)
- Suppose 5-D spacetime is (exponentially) *warped*, i.e. $d\mathfrak{s}^2 = e^{-2k\mathfrak{x}^4}\eta_{\mu\nu}dx^\mu dx^\nu + \left(d\mathfrak{x}^4\right)^2$.
- If $\mathfrak{x}^4$ is compact and of small size R, for a particle living on a brane at the warped end, the low-energy effective actionfor a scalar (for instance) is:

$$\begin{aligned} S_{4D} &\supset \int \left(g^{\mu\nu}D_\mu\phi^\dagger D_\nu\phi - \mathfrak{m}^2\left|\phi\right|^2\right)\sqrt{|g|}d^4x \\ &= \int \left(e^{2\pi Rk}\mathfrak{g}^{\mu\nu}D_\mu\phi^\dagger D_\nu\phi - \mathfrak{m}^2\left|\phi\right|^2\right)e^{-4\pi Rk}\sqrt{|\mathfrak{g}|}d^4x. \end{aligned}$$

- Canonical (re)normalizaion:

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- $m = e^{-\pi Rk}\mathfrak{m}. \Rightarrow$ **Hierarchy Problem solved**

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Warping

Non-Compact

- hep-ph/9906064: Randall & Sundrum (RS2)
- Take RS1, put us on the other brane, and take $R \to \infty$ limit
- Potential seen by graviton binds creates bound state at our brane
- Continuum of KK modes
- Coupling to massive KK modes supressed by $\frac{p}{k}$
- $V_g = G_N \frac{m_1 m_2}{r} \left(1 + \frac{1}{k^2 r^2}\right)$.
- Energy loss to bulk small
- **Cures a "moduli problem"** of string theory: runaway is OK

Duality and Strong Dynamics

AdS/CFT $\longrightarrow$ Technicolor **hep-th/0012148**

- Warped spacetimes are (slices of) anti de Sitter (AdS) spaces, having (-) curvature.
- hep-th/9711200: Maldacena duality
  - Quantum gravity on $AdS_{D+1} \leftrightarrow$ Large N Conformal Gauge Field Theory in D-dimensional spacetime (AdS/CFT)
- $\mathfrak{x}^4 \leftrightarrow$ RG scale
- Planck brane $\leftrightarrow$ UV cutoff
- RS2: Localization of graviton $\leftrightarrow$ 4D gravity
- RS1: TeV brane $\leftrightarrow$ breakdown of conformality in IR
- RS1: SM gauge bosons $\leftrightarrow$ bound states of broken CFT
- localizing a Higgs on TeV brane $\leftrightarrow$ bound state of broken CFT breaks EW (a.k.a. Technicolor)

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Universal Extra Dimensions

(UED)

- hep-ph/0012100:Applequist, H.-C. Cheng, & Dobrescu
- Allow everything to propagate in all 5-D
- Stronger constraits than LED on size:
  - EWPT
  - a_μ
  - FCNC's
- KK parity: Conservation of $\mathfrak{p}^4 \Rightarrow$ KK-modes annihilated/produced in pairs (or more)

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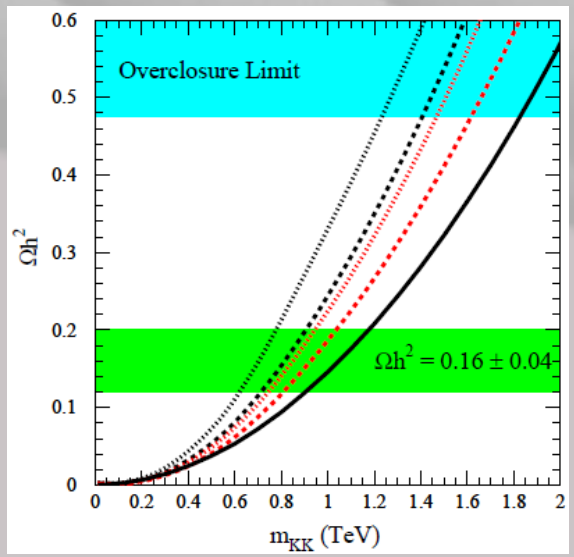
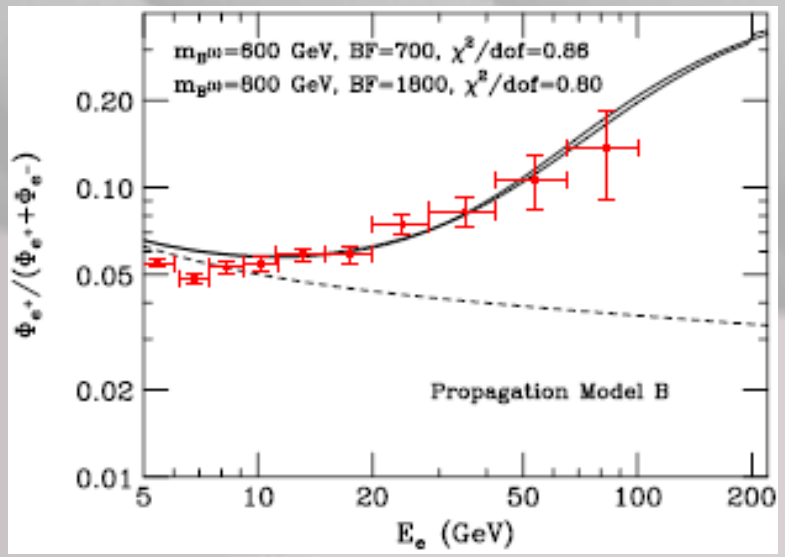
KK Parity

Dark Matter

- hep-ph/0204342: H.-C. Cheng, Matchev, & Schmaltz
- KK parity $\Rightarrow$ **stable particles**, possibly weak(ish)-scale, some without E&M/strong interactions
  $\Rightarrow$ Lightest KK particle (LKP) therefore potential **dark matter** (DM)
- Either KK γ or KK ν as LKP undergoing thermal freeze-out (FO) can get $\Omega_M \sim 0.3$. (Servant & Tait '02); also viable PAMELA explanation (Hooper & Zurek '09)





Other Consquences of KK Modes

- Possible new TeV-scale particles
- Non-compact extra dimensions $\Rightarrow$ possible missing momentum into bulk
- Could affect many SM processes at loop level (infinite towers)
  - ^(typically collider bounds stronger)

Black Holes

- In LED, fundamental scale is $\mathcal{O}(10\ \text{TeV})$.
- Collisions at this scale should form **black holes**!
- Short-lived due to rapid Hawking radiation
- Spectacular signal: isotropic (in rest frame), “democratic” decay
- Should be visible in (rare) high-energy cosmic rays

Motivation

In Summary

- **Why not?**
- Unification
- Quantizing gravity
- Justify fine tuning
- **Solve Hierarchy Problem**
- Natural **dark matter** candidates
- Equivalent to strong dynamics that may ~~**EW**~~
- **Dark matter**
- Spectacular signals